the value of information in multiagent coordination

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Central Goal

Derive **efficient** system-wide behavior through the design of **admissible** control algorithms

- **Sensor coverage**
- **Traffic network**
- **Smart Grid**
- **Socio-technical systems**
- **Engineering systems**
- **Social systems**
Multiagent coordination

Central Goal

Derive **efficient** system-wide behavior through the design of **admissible** control algorithms

Sensor coverage

**efficient**: desirable allocation for any initial conditions

**admissible**: local information, independent decisions

**challenge**: overlapping and partial information

engineering systems
**Central Goal**

Derive *efficient* system-wide behavior through the design of *admissible* control algorithms

*efficient*: desirable allocation for any network demands

*admissible*: local/aggregate incentive mechanism

*challenge*: uncontrollable entities, unknown sensitivity
Multiagent coordination

**Central Goal**

Derive *efficient* system-wide behavior through the design of *admissible* control algorithms

[Image: Multiagent coordination diagram showing sensor coverage, Smart Grid, socio-technical systems, engineering systems, and traffic network.]
Fundamental Limitations

- Constraints on admissible control laws
- Constraints on achievable performance

- Information
- Efficiency of behavior

- Sensor coverage
- Social systems

- Engineering systems
- Smart Grid

Multiagent coordination: socio-technical systems
Fundamental Limitations

- Constraints on admissible control laws
- Constraints on achievable performance

Information

Efficiency of behavior

What “information” should be shared between the agents to optimize efficiency of resulting collective behavior?
Multiagent coordination - Part I

**Fundamental Limitations**

- Constraints on admissible control laws
- Constraints on achievable performance

Information: local + communicated

Control: optimal decision rule given available information

Challenge: coupling between information + controls

What “information” should be shared between the agents to optimize efficiency of resulting collective behavior?
Fundamental Limitations

- Constraints on admissible control laws
- Constraints on achievable performance

What “information” is most valuable in the design of mechanisms for social influence?

Traffic network

Social systems
Fundamental Limitations

- Constraints on admissible control laws
- Constraints on achievable performance

What “information” is most valuable in the design of mechanisms for social influence?

- Uncertainty: infrastructure, population
- Control: incentive mechanisms to influence behavior
- Challenge: optimal incentives depends on information

Traffic network

Social systems
Fundamental Limitations

- Constraints on admissible control laws
- Constraints on achievable performance

What “information” should be shared between the agents to optimize efficiency of resulting collective behavior?

Sensor coverage

Engineering systems
**Motivating example**

**Goal:** Attain best information regarding targets

**Challenge:** Constraints on communication
Motivating example

Goal: Attain best information regarding targets

Challenge: Constraints on communication
**Goal:** Attain best information regarding targets

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Goal: Attain best information regarding targets

Challenge: Constraints on communication

Motivating example

command / fusion center

transmits limited # measurements

optimal selection is hard!

high number measurements
Motivating example

Goal: Attain best information regarding targets

Challenge: Constraints on communication

command / fusion center

quality depends on joint transmissions
Motivating example

**Goal:** Attain best information regarding targets

**Challenge:** Constraints on communication

What information do agents have? How should agents formulate choice?
Motivating example

**Goal:** Attain best information regarding targets

**Challenge:** Constraints on communication

What information do agents have? How should agents formulate choice?
Motivating example

Model

• Agents: \( N = \{1, 2, \ldots, n\} \)
• Choices: \( x_i \in X_i \)
• Evaluation: \( W(x_1 \cup \cdots \cup x_n) \)

What information do agents have?
How should agents formulate choice?
Motivating example

Model

- Agents: \( N = \{1, 2, \ldots, n\} \)
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- Evaluation: \( W(x_1 \cup \cdots \cup x_n) \)

Goal: Design admissible decision-making process

\[
x_i^{\text{sol}} = \Pi_i(\text{info available to } i)
\]

such that resulting collective choice is near optimal, i.e.,

\[
W(x_1^{\text{sol}} \cup \cdots \cup x_n^{\text{sol}}) \approx W(\text{OPT})
\]

command / fusion center

What information do agents have? How should agents formulate choice?
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\[ W(x_1^{\text{sol}} \cup \cdots \cup x_n^{\text{sol}}) \approx W(\text{OPT}) \]

Many positive results (information not explicit):

\[ \frac{W(\text{SOL})}{W(\text{OPT})} \geq \frac{1}{2} \]

What information do agents have? How should agents formulate choice?
Motivating example

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- **Agents:** \( N = \{1, 2, \ldots, n\} \)
- **Choices:** \( x_i \in X_i \)
- **Evaluation:** \( W(x_1 \cup \cdots \cup x_n) \)

**Example:** Greedy Algorithm

- **Information:** \( i \Rightarrow x_{1}^{\text{sol}}, \ldots, x_{i-1}^{\text{sol}} \)
- **Choice rule:** \( x_{i}^{\text{sol}} \in \arg \max_{x_i \in X_i} W(x_i \cup x_{1}^{\text{sol}} \cup \cdots \cup x_{i-1}^{\text{sol}}) \)
Model

- Agents: \( N = \{1, 2, \ldots, n\} \)
- Choices: \( x_i \in X_i \)
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Representative theorems: If \( W \) has nice properties, then we have nice guarantees
Motivating example

Model

- Agents: \( N = \{1, 2, \ldots, n\} \)
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Example: Greedy Algorithm

- Information: \( i \Rightarrow x_1^{sol}, \ldots, x_{i-1}^{sol} \)
- Choice rule: \( x_i^{sol} \in \arg \max_{x_i \in X_i} W(x_i \cup x_1^{sol} \cup \cdots \cup x_{i-1}^{sol}) \)

Submodularity: Decreasing marginal returns

\[
W(A \cup \{x\}) - W(A) \geq W(B \cup \{x\}) - W(B)
\]

marginal gain adding \( x \) to “smaller” set \( A \)
marginal gain adding \( x \) to “larger” set \( B \)
Motivating example

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- Choices: \( x_i \in X_i \)
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Example: Greedy Algorithm

- Information: \( i \Rightarrow x_1^{sol}, \ldots, x_{i-1}^{sol} \)
- Choice rule: \( x_i^{sol} \in \arg \max_{x_i \in X_i} W(x_i \cup x_1^{sol} \cup \cdots \cup x_{i-1}^{sol}) \)

Submodularity: Decreasing marginal returns

Theorem: If \( W \) is submodular, then greedy algorithm guarantees

\[
\frac{W(SOL)}{W(OPT)} \geq \frac{1}{2}
\]
Motivating example

Model
- Agents: $N = \{1, 2, \ldots, n\}$
- Choices: $x_i \in X_i$
- Evaluation: $W(x_1 \cup \cdots \cup x_n)$

Example: Greedy Algorithm
- Information: $i \Rightarrow x_{1}^{\text{sol}}, \ldots, x_{i-1}^{\text{sol}}$
- Choice rule: $x_{i}^{\text{sol}} \in \arg \max_{x_i \in X_i} W(x_i \cup x_1^{\text{sol}} \cup \cdots \cup x_{i-1}^{\text{sol}})$

Submodularity: Decreasing marginal returns

Theorem: If $W$ is submodular, then greedy algorithm guarantees

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\frac{W(\text{SOL})}{W(\text{OPT})} \geq \frac{1}{2}
\]

Issues:
- Informational demand?
- Complexity of choice rule?
Model

- Agents: \( N = \{1, 2, \ldots, n\} \)
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- Evaluation: \( W(x_1 \cup \cdots \cup x_n) \)

Example: Greedy Algorithm

- Information: \( i \Rightarrow x_1^{sol}, \ldots, x_{i-1}^{sol} \)
- Choice rule: \( x_i^{sol} \in \arg \max_{x_i \in X_i} W(x_i \cup x_1^{sol} \cup \cdots \cup x_{i-1}^{sol}) \)

\( (best\ response) \)

\( (approximate\ best\ response) \)

\[ W(x_i^{sol} \cup x_1^{sol}) \geq \alpha \cdot \max_{x_i \in X_i} W(x_i \cup x_1^{sol}) , \alpha \in [0, 1] \]
Motivating example

Model

- **Agents:** \( N = \{1, 2, \ldots, n\} \)
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- **Evaluation:** \( W(x_1 \cup \cdots \cup x_n) \)

Example: Greedy Algorithm

- **Information:** \( i \Rightarrow x_1^{sol}, \ldots, x_{i-1}^{sol} \)
- **Choice rule:** \( x_i^{sol} \in \arg \max_{x_i \in X_i} W(x_i \cup x_1^{sol} \cup \cdots \cup x_{i-1}^{sol}) \) (best response)

\[ \begin{align*}
W(x_i^{sol} \cup x_{1:i-1}^{sol}) &\geq \alpha \cdot \max_{x_i \in X_i} W(x_i \cup x_{1:i-1}^{sol}) , \quad \alpha \in [0, 1]
\end{align*} \]

Observation: Local approximations do not significantly degrade solution quality

\[ \begin{align*}
\frac{W(SOL)}{W(OPT)} &\geq \frac{1}{2} & \quad & \frac{W(SOL)}{W(OPT)} &\geq \frac{1}{1 + \frac{1}{\alpha}}
\end{align*} \]
Motivating example

Model

- **Agents:** \(N = \{1, 2, \ldots, n\}\)
- **Choices:** \(x_i \in X_i\)
- **Evaluation:** \(W(x_1 \cup \cdots \cup x_n)\)

Example: Greedy Algorithm

- **Information:** \(i \Rightarrow x_{1}^{\text{sol}}, \ldots, x_{i-1}^{\text{sol}}\)
- **Choice rule:** \(x_{i}^{\text{sol}} \in \text{arg max}_{x_i \in X_i} W(x_i \cup x_{1}^{\text{sol}} \cup \cdots \cup x_{i-1}^{\text{sol}})\)

Submodularity: Decreasing marginal returns

Theorem: If \(W\) is submodular, then greedy algorithm guarantees

\[
\frac{W(\text{SOL})}{W(\text{OPT})} \geq \frac{1}{2}
\]

Issues:

- Informational demand? **FOCUS**
- Complexity of choice rule?
Goal: Design admissible decision-making process

\[ x_i^{sol} = \Pi_i(\text{info available to } i) \]

such that resulting collective choice is near optimal, i.e.,

\[ W(x_1^{sol} \cup \cdots \cup x_n^{sol}) \approx W(\text{OPT}) \]

Two design questions:

- What information should agents share?
- What should agents do with information?

Focus: “Solution” quality relative to OPT

Our result: Optimal information exchange in limited context
Augmented greedy algorithm

Model

- Agents: \( N = \{1, 2, \ldots, n\} \)
- Choices: \( x_i \in X_i \)
- Evaluation: \( W(x_1 \cup \cdots \cup x_n) \)

Example: Greedy Algorithm

- Information: \( i \Rightarrow x_1^{sol}, \ldots, x_{i-1}^{sol} \)
- Choice rule: \( x_i^{sol} \in \arg\max_{x_i \in X_i} W(x_i \cup x_1^{sol} \cup \cdots \cup x_{i-1}^{sol}) \)
Augmented greedy algorithm

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- **Agents:**  \( N = \{1, 2, \ldots, n\} \)
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**Definition:** Augmented Greedy Algorithm
Augmented greedy algorithm

Model
- **Agents:** \( N = \{1, 2, \ldots, n\} \)
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Definition: Augmented Greedy Algorithm
- **Information:** \( i \Rightarrow x_1^{sol}, \ldots, x_{i-1}^{sol}, z_1^{sol}, \ldots, z_{i-1}^{sol} \)
Augmented greedy algorithm

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- Agents: \( N = \{1, 2, \ldots, n\} \)
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Augmented greedy algorithm

Model

- **Agents:** \( N = \{1, 2, \ldots, n\} \)
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Example: Greedy Algorithm

- **Information:** \( i \Rightarrow x_{i-1}^{sol}, \ldots, x_1^{sol} \)
- **Choice rule:** \( x_i^{sol} \in \arg\max_{x_i \in X_i} W(x_i \cup x_1^{sol} \cup \cdots \cup x_{i-1}^{sol}) \)

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- **Information:** \( i \Rightarrow x_{i-1}^{sol}, z_{i-1}^{sol}, \ldots, x_1^{sol}, z_1^{sol} \)
- **Communication rule:** \( z_i^{sol} = \prod_{i}^{\text{comm}} (x_{i-1}^{sol}, x_1^{sol}, z_1^{sol}, \ldots, z_{i-1}^{sol}) \)
- **Choice rule:** \( x_i^{sol} = \prod_{i}^{\text{choice}} (x_{i-1}^{sol}, x_1^{sol}, z_1^{sol}, z_{i-1}^{sol}) \)
Augmented greedy algorithm

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- Agents: $N = \{1, 2, \ldots, n\}$
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Example: Greedy Algorithm

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- Choice rule: $x_i^{sol} = \Pi_i^{\text{choice}} (x_1^{sol}, \ldots, x_{i-1}^{sol}, z_{1}^{sol}, \ldots, z_{i-1}^{sol})$

Goal: Design communication rule + choice rule to optimize performance guarantees
Augmented greedy algorithm

**Definition:** Augmented Greedy Algorithm

- **Information:**
  \[ i \Rightarrow x_{1}^{\text{sol}}, \ldots, x_{i-1}^{\text{sol}}, z_{1}^{\text{sol}}, \ldots, z_{i-1}^{\text{sol}} \]

- **Communication rule:**
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**Augmented greedy algorithm**

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**Augmented greedy algorithm**

**Definition:** Augmented Greedy Algorithm

- **Information:**
  \[ i \Rightarrow x^{sol}_1, \ldots, x^{sol}_{i-1}, z^{sol}_1, \ldots, z^{sol}_{i-1} \]

- **Communication rule:**
  \[ z^{sol}_i = \prod_i^{\text{comm}} (x^{sol}_1, \ldots, x^{sol}_{i-1}, z^{sol}_1, \ldots, z^{sol}_{i-1}) \]

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Augmented greedy algorithm

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• Choice rule:
  \[ x_{i}^{\text{sol}} = \prod_{i}^{\text{choice}} (x_{1}^{\text{sol}}, \ldots, x_{i-1}^{\text{sol}}, z_{1}^{\text{sol}}, \ldots, z_{i-1}^{\text{sol}}) \]

Diagram:

1. Selection
2. Communication
3. Command / Fusion center
**Augmented greedy algorithm**

**Definition:** Augmented Greedy Algorithm

- **Information:**
  \[ i \Rightarrow x^\text{sol}_1, \ldots, x^\text{sol}_{i-1}, z^\text{sol}_1, \ldots, z^\text{sol}_{i-1} \]

- **Communication rule:**
  \[ z^\text{sol}_i = \prod^\text{comm}_i (x^\text{sol}_1, \ldots, x^\text{sol}_{i-1}, z^\text{sol}_1, \ldots, z^\text{sol}_{i-1}) \]

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Definition: Augmented Greedy Algorithm

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**Constraints:**

- **Communication:**
  \[ |z_{i}^{\text{sol}}| \leq k_{1}, z_{i}^{\text{sol}} \subseteq X_{i} \]

- **Choices:**
  \[ |x_{i}^{\text{sol}}| \leq k_{2}, x_{i}^{\text{sol}} \subseteq X_{i} \cup z_{1}^{\text{sol}} \cup \cdots \cup z_{i-1}^{\text{sol}} \]
**Augmented greedy algorithm**

**Definition:** Augmented Greedy Algorithm

- Information: \( i \Rightarrow x_1^\text{sol}, \ldots, x_{i-1}^\text{sol}, z_1^\text{sol}, \ldots, z_{i-1}^\text{sol} \)
- Communication rule: \( z_i^\text{sol} = \prod_i^{\text{comm}} (x_1^\text{sol}, \ldots, x_{i-1}^\text{sol}, z_1^\text{sol}, \ldots, z_{i-1}^\text{sol}) \)
- Choice rule: \( x_i^\text{sol} = \prod_i^{\text{choice}} (x_1^\text{sol}, \ldots, x_{i-1}^\text{sol}, z_1^\text{sol}, \ldots, z_{i-1}^\text{sol}) \)

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- Communication: \(|z_i^\text{sol}| \leq k_1, z_i^\text{sol} \subseteq X_i\)
- Choices: \(|x_i^\text{sol}| \leq k_2, x_i^\text{sol} \subseteq X_i \cup z_1^\text{sol} \cup \cdots \cup z_{i-1}^\text{sol}\)

**Question:** What is optimal communication rule, choice rule, and performance guarantees?
Augmented greedy algorithm

**Definition:** Augmented Greedy Algorithm

- **Information:** $i \Rightarrow x_{i_1}^{sol}, \ldots, x_{i-1}^{sol}, z_1^{sol}, \ldots, z_{i-1}^{sol}$
- **Communication rule:** $z_i^{sol} = \prod_{i}^{comm} (x_1^{sol}, \ldots, x_{i-1}^{sol}, z_1^{sol}, \ldots, z_{i-1}^{sol})$
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**Constraints:**

- **Communication:** $|z_i^{sol}| \leq k_1, z_i^{sol} \subseteq X_i$
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**Question:** What is optimal communication rule, choice rule, and performance guarantees?

**Theorem:** [Grimsman et al., 2020]: If system objective is submodular, then

$$\frac{1}{2 - \frac{\min(k_1/k_2, 1)}{n-1+\min(k_1/k_2, 1)}} \geq \text{Performance Guarantees}(k_1, k_2) \geq \frac{1}{2 - \frac{\sum_{i=0}^{n-1} (\min(k_1/k_2, 1))^i}{n-1+\min(k_1/k_2, 1)^{n-1}}}$$

(impossibility) (realizable)
Augmented greedy algorithm

Observations:

- $k_1/k_2$ driving factor of performance guarantees

Constraints:

- Communication: $|z_i^{sol}| \leq k_1, z_i^{sol} \subseteq X_i$
- Choices: $|x_i^{sol}| \leq k_2, x_i^{sol} \subseteq X_i \cup z_1^{sol} \cup \cdots \cup z_{i-1}^{sol}$

Theorem: [Grimsman et al., 2020]: If system objective is submodular, then

\[
2 - \frac{1}{\min(k_1/k_2, 1)} \geq \text{Performance Guarantees}(k_1, k_2) \geq \frac{1}{2 - \sum_{i=0}^{n-1} (\min(k_1/k_2, 1))^i}
\]

(impossibility) (realizable)
Augmented greedy algorithm

Observations:

• $k_1/k_2$ driving factor of performance guarantees
• It is never beneficial relative to performance bounds to have $k_1 > k_2$

Constraints:

• Communication: $|z_i^{\text{sol}}| \leq k_1$, $z_i^{\text{sol}} \subseteq X_i$
• Choices: $|x_i^{\text{sol}}| \leq k_2$, $x_i^{\text{sol}} \subseteq X_i \cup z_1^{\text{sol}} \cup \cdots \cup z_{i-1}^{\text{sol}}$

Theorem: [Grimsman et al., 2020]: If system objective is submodular, then

\[
2 - \frac{1}{\min(k_1/k_2,1)^{n-1} + \min(k_1/k_2,1)} \geq \text{Performance Guarantees}(k_1, k_2) \geq 2 - \frac{1}{\sum_{i=0}^{n-1} (\min(k_1/k_2,1))^i}
\]

(impossibility) \hspace{1cm} (realizable)
Augmented greedy algorithm

Observations:

- \( \frac{k_1}{k_2} \) driving factor of performance guarantees
- It is never beneficial relative to performance bounds to have \( k_1 > k_2 \)
- Table shows % increase in performance guarantees relative to greedy algorithm

\[
\begin{align*}
\text{Performance Guarantees}(k_1, k_2) &\geq \frac{1}{2 - \frac{1 - \min(k_1/k_2, 1)}{n-1 + \min(k_1/k_2, 1)}} \\
\text{Theorem: [Grimsman et al., 2020]} &\quad \text{If system objective is submodular, then}
\end{align*}
\]

<table>
<thead>
<tr>
<th>( \frac{k_1}{k_2} )</th>
<th>( n = 2 )</th>
<th>( n = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Impossibility</td>
<td>Realizable</td>
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<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.2</td>
<td>9.09%</td>
<td>9.09%</td>
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<tr>
<td>0.4</td>
<td>16.67%</td>
<td>16.67%</td>
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<tr>
<td>0.6</td>
<td>23.08%</td>
<td>23.08%</td>
</tr>
<tr>
<td>0.8</td>
<td>28.57%</td>
<td>28.57%</td>
</tr>
<tr>
<td>1.0</td>
<td>33.33%</td>
<td>33.33%</td>
</tr>
</tbody>
</table>

\( k_1 \) and \( k_2 \) are the driving factors of performance guarantees. The table shows the % increase in performance guarantees relative to greedy algorithm as \( \frac{k_1}{k_2} \) varies. The theorem ensures that if the system objective is submodular, then the performance guarantees are bounded.
Simulation

- two agents, two targets
- random placement and trajectories
- $k_1 = k_2 = 2$
- Evaluation = Error covariance (logdet)
- 1 million trials

\[
\frac{W(\text{Solution Augmented Greedy})}{W(\text{Solution Standard Greedy})}
\]

- Gray = Communication not used
- Blue = One communication used
- Orange = Two communications used

<table>
<thead>
<tr>
<th>Metric</th>
<th>% Sim</th>
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<tbody>
<tr>
<td>Performance increase</td>
<td>46.12%</td>
</tr>
<tr>
<td>Performance decrease</td>
<td>0.0003%</td>
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<tr>
<td>Performance increase at least 5%</td>
<td>35.10%</td>
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<tr>
<td>Performance increase at least 40%</td>
<td>20.36%</td>
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<tr>
<td>Performance increase at least 70%</td>
<td>16.05%</td>
</tr>
<tr>
<td>Average</td>
<td>1.178</td>
</tr>
</tbody>
</table>
constraints on admissible control laws

information

constraints on achievable performance

efficiency of behavior

What “information” is most valuable in design of mechanisms for social influence?

self-interested behavior not optimal

incentives influence self-interested behavior

optimal incentives depends on information
Multiagent coordination - Part II

Constraints on admissible control laws

Constraints on achievable performance

Information

Efficiency of behavior

What “information” is most valuable in design of mechanisms for social influence?

Infrastructure

Population

Network structure?

Edge characteristics?

Users demands?

Monetary sensitivities?

What is the value of information with regards to social coordination?

(more is better, but how much better?)
Non-atomic congestion games

Feasible flows: \( f = \{ f_e \}_{e \in E} \in \mathcal{F} \)

Congestion functions: \( c_e : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \)

System-level objective: \( C(f) = \sum_{e \in E} f_e \cdot c_e(f_e) \)
**Non-atomic congestion games**

**Assumption:** Nash flows represent reasonable description of behavior

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\text{Feasible flows:} & \quad f = \{f_e\}_{e \in E} \in \mathcal{F} \\
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- Uninfluenced: \( J_i(f) = \sum_{e \in a_i} c_e(f_e) \)
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Assumption: Nash flows represent reasonable description of behavior

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- Influenced: \( J_i(f) = \sum_{e \in a_i} c_e(f_e) + s_i \cdot t_e(f_e) \)

unknown sensitivity designed taxation
Assumption: Nash flows represent reasonable description of behavior

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- Influenced: 
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Goal: Given informational knowledge, design taxation mechanism to optimize POA
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Goal: Given informational knowledge, design taxation mechanism to optimize POA

*(Dario Paccagnan will discuss approaches to optimal taxation design tomorrow)*
Non-atomic congestion games

Feasible flows:
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Congestion functions:
\[ c_e : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \]

System-level objective:
\[ C(f) = \sum_{e \in E} f_e \cdot c_e(f_e) \]

Theorem [Ferguson et al., 2020]

Objective function:
\[ \text{OBJ} = \min \frac{C(\text{Nash Flow})}{C(\text{OPT})} \]

Optimal taxation mechanisms:
\[ \{t_e\}_{e \in E} \]

(limited class of networks)
Non-atomic congestion games

Feasible flows:
\[ f = \{ f_e \}_{e \in E} \in \mathcal{F} \]

Congestion functions:
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No Information |  |  | Complete Information
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**better network knowledge**

Complete Information
Non-atomic congestion games

Feasible flows: $f = \{f_e\}_{e \in E} \in \mathcal{F}$
Congestion functions: $c_e : \mathbb{R}^+ \rightarrow \mathbb{R}^+$
System-level objective: $C(f) = \sum_{e \in E} f_e \cdot c_e(f_e)$

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Non-atomic congestion games

Feasible flows: \( f = \{f_e\}_{e \in E} \in \mathcal{F} \)
Congestion functions: \( c_e : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \)
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\[
\frac{C(\text{Nash Flow})}{C(\text{Optimal Flow})}
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<td></td>
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(population)

(two link affine congestion game, \( s = 1, \bar{s} = 10 \))
Feasible flows: 
\[ f = \{ f_e \}_{e \in E} \in \mathcal{F} \]
Congestion functions: 
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<td>[ C(\text{Nash Flow})/C(\text{Optimal Flow}) ]</td>
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<tr>
<td><strong>agnostic</strong></td>
<td>4/3 [ \text{[Roughgarden]} ]</td>
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<td></td>
<td>1 [ \text{[Fleischer]} ]</td>
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(two link affine congestion game, \( \underline{s} = 1, \overline{s} = 10 \))
Non-atomic congestion games

Feasible flows: \( f = \{ f_e \}_{e \in E} \in \mathcal{F} \)

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<tbody>
<tr>
<td>4/3 [Roughgarden]</td>
<td>1.176 [Brown]</td>
<td></td>
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</table>

| aware | |
|-------|
| 1 [Fleischer] |

(two link affine congestion game, \( \underline{s} = 1, \overline{s} = 10 \))
Non-atomic congestion games

Feasible flows: \[ f = \{f_e\}_{e \in E} \in \mathcal{F} \]
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Population

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<td>4/3</td>
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<tr>
<td>[Roughgarden]</td>
<td>[Brown]</td>
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Non-atomic congestion games

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\[
\begin{array}{|c|c|c|c|}
\hline
\text{Population} & \text{Agnostic} & \text{Limited} & \text{Aware} \\
\hline
\text{Agnostic infrastructure} & \frac{4}{3} & 1.176 & 1.1401 \\
 & \text{[Roughgarden]} & \text{[Brown]} & \\
\hline
\text{Aware} & & & 1 \\
 & & \text{[Fleischer]} & \\
\hline
\end{array}
\]

(two link affine congestion game, \( s = 1, \overline{s} = 10 \) )
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<td>1.1401</td>
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\text{population} & \text{agnostic} & \text{limited} & \text{aware} \\
\hline
\text{agnostic infrastructure} & \frac{4/3}{4/3} & 1.176 & 1.1401 & 1.1401 \\
\text{aware} & 4/3 & 1.1401 & \text{1} & \text{1} \\
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<td>( 4/3 )</td>
<td>1.09</td>
<td>1.0494</td>
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(two link affine congestion game, \( s = 1, \bar{s} = 10 \))
Non-atomic congestion games

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### table informs us about where to invest in getting better information

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data informs us about where to invest in getting better information

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<td>( C(\text{Nash Flow}) / C(\text{Optimal Flow}) )</td>
<td>( s_i \geq 0 )</td>
<td>( s_i \in [\underline{s}, \overline{s}] )</td>
<td>( s_i \in [\underline{s}, \overline{s}], s_{\text{mean}} )</td>
</tr>
<tr>
<td>agnostic</td>
<td>4/3 [\text{Roughgarden}]</td>
<td>1.176 [\text{Brown}]</td>
<td>?</td>
</tr>
<tr>
<td>aware</td>
<td>4/3</td>
<td>1.09</td>
<td>1.0494</td>
</tr>
</tbody>
</table>

(two link affine congestion game, \( \underline{s} = 1, \overline{s} = 10 \))
**Conclusion**

**Fundamental Limitations**

- Constraints on admissible control laws
- Information

- Constraints on achievable performance
- Efficiency of behavior

What is the value of information in multiagent coordination?

**Sensor coverage**

more is better….

but how much?

prioritization of information?

is more always better?

**Traffic network**
Highlighted papers:


Relevant Papers: